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## Abstract

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## Keywords

RIKEN BNL Research Center

## Disciplines

Astrophysics and Astronomy | Physics

## Comments

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# Forward hadron production in high energy pA collisions: from RHIC to LHC

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(Dated: February 17, 2013)

We present a calculation of inclusive  $\pi$ ,  $D$  and  $B$  mesons production at RHIC and LHC energies based upon the KKT model of gluon saturation. We discuss dependence of the nuclear modification factor on rapidity, centrality and transverse momentum.

## I. INTRODUCTION

In this paper we use the phenomenological model of Ref. [1] (KKT model) to analyze inclusive hadron production in pA collisions at RHIC and LHC. We focus on energy, rapidity, centrality and transverse momentum dependence of the nuclear modification factor  $R_{pA}$  ( $p$  stands for either proton or deuteron) for  $\pi$ ,  $D$  and  $B$  meson production. Theoretical analysis [2, 3, 4, 5] reveals that coherent effects in hadron production become phenomenologically significant at RHIC. They consists of (in the nucleus rest frame): (i) multiple rescattering of the incoming parton system in the nucleus. This is controlled by the parameter  $\alpha_s^2 A^{1/3} \sim 1$ . And (ii) small- $x$  quantum evolution controlled by the parameter  $\alpha_s y = \alpha_s \ln(1/x) \sim 1$ . Both effects lead to the gluon saturation of a dense parton system at small  $x$  which manifests itself in a different ways depending on the kinematic region we are interested in: (i) at not too small  $x$  ( $e^{-1/\alpha_s} < x \ll 1$ ) it contributes to the Cronin effect as the result of multiple rescatterings of the projectile in the target; (ii) at  $x \lesssim e^{-1/\alpha_s}$  the small- $x$  evolution starts off which results in a dramatic suppression of  $R_{pA}^\pi$  at forward rapidities at RHIC. Similar

effects have been predicted [6, 7, 8, 9, 10] and observed [11] for open charm production. It is of great interest to observe the evolution of these phenomena with energy. Thus, motivated by the approaching start of the nuclear program at LHC we decided to combine calculations of inclusive hadron production in a framework of a single model.

The paper is structured as follows. In Sec. II we review a phenomenological model for the dipole scattering amplitude  $N(\underline{r}, y)$  [1] which has been previously used to successfully describe the data on light hadron production at RHIC. Although  $N(\underline{r}, y)$  can in principle be analytically calculated using the small- $x$  evolution equation [12, 13], it's a very difficult and not yet solved problem. In addition, there is a fair amount of uncertainty in how the NLO terms in small- $x$  evolution change the behavior of  $N(\underline{r}, y)$  as well as in the effect of fluctuations in the dense partonic system. On the other hand, rather general arguments allow identification of a few key features which hold for  $N(\underline{r}, y)$  in a dense parton system. These features are heeded in constructing a phenomenological model for  $N(\underline{r}, y)$ . In this perspective we argue in favor of the KKT model of Ref. [3].

In Sec. III and Sec. IV we present theoretical results which together with the model of Sec. II enables calculation of pion and open charm and beauty production at RHIC and LHC. These results are summarized in figures Fig. 1, Fig. 3 and Fig. 4. They teach us that at LHC at rapidities  $y \geq 0$ , the nuclear modification factor is a very slow function of rapidity. This is an anticipated result. Indeed, the amount of suppression can be estimated as  $R_{pA} \simeq 1/N_{\text{coll}}^{1-\gamma}$ , where  $\gamma$  depends on rapidity and transverse momentum [2, 3]. Eqs. (2) and (17) imply that  $\gamma$  decreases with energy and rapidity and increases with transverse momentum and mass, although it is a very slow function of its variables. Thus, we observe almost the same suppression pattern as the function of transverse momentum, rapidity and centrality for gluons and heavy quarks. To emphasize the mass and transverse momentum dependence of  $R_{pA}$  we plotted it versus  $m$  in Fig. 5 up to the top-quark mass. Despite that  $x$  is small in the kinematic region of this figure, the geometric scaling and hence the KKT model, are expected to break down. Still there is a vague theoretical understanding of how exactly it occurs. This too is discussed in Sec. IV.

## II. A MODEL

In the dipole model [14], cross sections for inclusive hadron production can be expressed through the gluon dipole forward scattering amplitude  $N(\underline{r}, y)$ , where  $\underline{r}$  is the transverse separation and  $y$  is rapidity. This quantity can be calculated for any  $\underline{r}$  and  $y > 1$  using the small- $x$  evolution equations, the most useful of which is the BK equation[12, 13]. In order to be able to utilize an exact numerical solution to BK [15, 16, 17, 18, 19, 20] one needs to (i) include the NLO corrections into the BK equation and (ii) perform global analysis of all small  $x$  data. Until this program is carried out, the most practical way to proceed is to use a model for  $N(\underline{r}, y)$  which has the most essential properties of solution to the BK equation. Several such models have been suggested [1, 21, 22, 23, 24, 25, 26]. Although they are not very much different from one another, the recent analysis of the numerical solution to BK equation performed in Ref. [27] seems to imply that the KKT-like models are closer to the numerical solution than the other models. The main observation of Ref. [27] is that the anomalous dimension  $\gamma$  at the saturation scale is about 0.44. This should be compared to the anomalous dimension of 0.5 used by models based on the double logarithmic approximation, such as KKT, and to the anomalous dimension of 0.628 used by models based on the saddle-point approximation. It should be kept in mind that  $\gamma$  is the most important parameter determining the maximal possible suppression of the nuclear modification factor due to the gluon saturation effect. On the other hand, the KKT model is certainly oversimplified and, as more accurate data appear, it will be replaced by a more realistic models.

For the reasons described above and encouraged by success of the KKT model in describing inclusive light hadron production at RHIC we set to employ it for calculation of inclusive hadron production at LHC. According to Ref. [1] the forward gluon dipole scattering amplitude is parameterized as follows

$$N(\underline{r}, y) = 1 - \exp \left\{ -\frac{1}{4} (r^2 Q_s^2)^{\gamma(r, y)} \right\}, \quad (1)$$

where we denoted  $|\underline{r}| \equiv r$ . The anomalous dimension is parameterized in such a way as to satisfy the analytically well-known limits of (i)  $r \rightarrow 0$ ,  $y$  fixed and (ii)  $y \rightarrow \infty$ ,  $r$  fixed:

$$\gamma(r, y) = \begin{cases} \frac{1}{2} \left( 1 + \frac{\xi(r, y)}{|\xi(r, y)| + \sqrt{2}|\xi(r, y)| + 28\zeta(3)} \right) & y \geq y_0, \\ 1 & y < y_0, \end{cases} \quad (2)$$

where

$$\xi(r, y) = \frac{\ln [1/(r^2 Q_{s0}^2)]}{(\lambda/2)(y - y_0)}. \quad (3)$$

In the double logarithmic approximation we can replace  $\underline{r}^2 \approx 1/(4\underline{k}^2)$ . The *gluon* saturation scale is given by

$$Q_s^2(y) = \Lambda^2 A^{1/3} e^{\lambda y} = 0.13 \text{ GeV}^2 e^{\lambda y} N_{\text{coll}}. \quad (4)$$

Parameters  $\Lambda = 0.6 \text{ GeV}$  and  $\lambda = 0.3$  are fixed by DIS data [21]. The minimal saturation scale used in (3) is defined by  $Q_{s0}^2 = Q_s^2(y_0)$  with  $y_0$  the value of rapidity at which the small- $x$  quantum evolution effects set in. Fit to the RHIC data yields  $y_0 = 0.5$  [1]<sup>1</sup>.

In the quasi-classical approximation (at  $\gamma = 1$ ) the forward scattering amplitude of the quark dipole  $N_Q(\underline{r}, y)$  is specified by the same formula (1) with the gluon saturation scale rescaled by the color factor  $C_F/N_c$ . Therefore, we model  $N_Q(\underline{r}, y)$  in the whole kinematic region by

$$N_Q(\underline{r}, y) = 1 - \exp \left\{ -\frac{1}{4} (r^2 Q_s^2/2)^{\gamma(r, y)} \right\}, \quad (5)$$

where in the large  $N_c$  limit  $C_F/N_c = 1/2$ .

### III. LIGHT HADRON PRODUCTION

The light hadron production cross section is a sum of two terms: inclusive gluon and valence quark production and hadronization. Inclusive gluon production dominates in most of the kinematic region safe for the most forward rapidities in the proton/deuteron fragmentation region, where Bjorken  $x$  of nucleus ( $x_A$ ) acquires its lowest possible value for a given  $\sqrt{s}$ , while that of proton/deuteron ( $x_p$ ) is close to unity. In that region rescattering of valence quarks of proton in nucleus dominate to the hadron production cross section.

The cross section for inclusive gluon production is given by

$$\frac{d\sigma_G}{d^2k dy} = \frac{\alpha_s C_F S_A}{\pi^2 k^2} x_p^{-\lambda} (1 - x_p)^4 \int_0^\infty dz_T J_0(k_T z_T) \ln \frac{1}{z_T \mu} \partial_{z_T} [z_T \partial_{z_T} N_G(z_T, y)], \quad (6)$$

where  $\mu$  is a scale associated with deuteron and is fixed at  $\mu = 1 \text{ GeV}$  thereof. Expression  $x_p^{-\lambda} (1 - x_p)^4$  is a model for the gluon pdf of proton. Inclusive valence quark production cross

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<sup>1</sup> In [1] it has been argued that a non-perturbative scale (“intrinsic  $k_T$ ”) which is responsible for the Cronin effect at low energies has a little importance at RHIC. It is therefore neglected in this paper.

section [28]

$$\frac{d\sigma_Q}{d^2k} = \frac{S_A}{2\pi} \int_0^\infty dz_T z_T J_0(k_T z_T) [2 - N_Q(z_T, y)], \quad (7)$$

The nuclear modification factor is defined as

$$R_{pA}(k, y) = \frac{\frac{d\sigma^h(pA)}{d^2k dy}}{A \frac{d\sigma^h(pp)}{d^2k dy}} = \frac{\frac{dN(pA)}{d^2k dy}}{N_{\text{coll}} \frac{dN(pp)}{d^2k dy}}, \quad (8)$$

where  $\frac{dN(pA)}{d^2k dy}$  and  $\frac{dN(pp)}{d^2k dy}$  are multiplicities of hadrons per unit of phase space in pA and pp collisions. Both expressions for gluon (6) and valence quark (7) production contribute to the hadron production cross section in pA and pp collisions. The cross section of hadron production reads

$$\begin{aligned} \frac{d\sigma^h}{d^2k dy} &= \int \frac{dz}{z^2} \frac{d\sigma_G}{d^2k dy}(k/z) D_G(z, k) F(k/z, y) \\ &+ \int \frac{dz}{z^2} \frac{d\sigma_Q}{d^2k}(k/z) xq_V(y, k/z) D_Q(z, k_T) F(k/z, y). \end{aligned} \quad (9)$$

We use the LO fragmentation functions from Ref. [29] with the renormalization scale of the fragmentation functions equal  $k$ .

Equation (7) is derived for production of a valence quark in the proton/deuteron fragmentation region. To generalize it to smaller values of Bjorken  $x$  one has to convolute it with the proton's/deuteron's valence quark distribution, which is fixed by quark counting rules at high  $x$  and by the leading Regge trajectory at low  $x$

$$xq_V(x) = 1.09 (1 - x_p)^3 x_p^{0.5}, \quad (10)$$

where  $x_p = (k_T/\sqrt{s}) e^\eta$ . Valence quarks are increasingly less important at low  $x$  [36], where the quark production is dominated by gluons splitting in  $q\bar{q}$  pairs. The factor of  $x_p^{0.5}$  insures that this is indeed the case here [36]. Analogously, the high  $x$  behavior of the *nuclear* gluon distribution is taken into account by introducing the function  $F(k, y)$

$$F(k, y) = (1 - x_A)^4 \left( \frac{\Lambda^2}{k^2 + \Lambda^2} \right)^{1.3\alpha_s}. \quad (11)$$

where the Bjorken  $x$  of a gluon in the nuclear wave function is given by  $x_A = (k/\sqrt{s}) e^{-\eta}$  and  $\alpha_s = 0.3$ . The last factor in Eq. (11) arises when we impose momentum conservation constraint on the anomalous dimension of the distribution functions.

Eq. (11) is an empiric way to include the higher order pQCD corrections. It is well-known that the NLO pQCD corrections are required to describe the inclusive  $\pi^0$  production in pp

collisions at RHIC [30, 31]. Therefore, these corrections must be taken into account both in the numerator and the denominator of Eq. (8). Unfortunately, the theoretical work on inclusion of the NLO corrections in the gluon saturation regime proved to be a very laborious problem. Only recently, calculations have been done for inclusion of some running coupling corrections into the forward scattering amplitude [32, 33, 34, 35]. This is the main reason why presently, we have to resort to the phenomenological approaches to the NLO corrections such as the one described here.

The results of numerical calculations are exhibited in Fig. 1. As has been already pointed

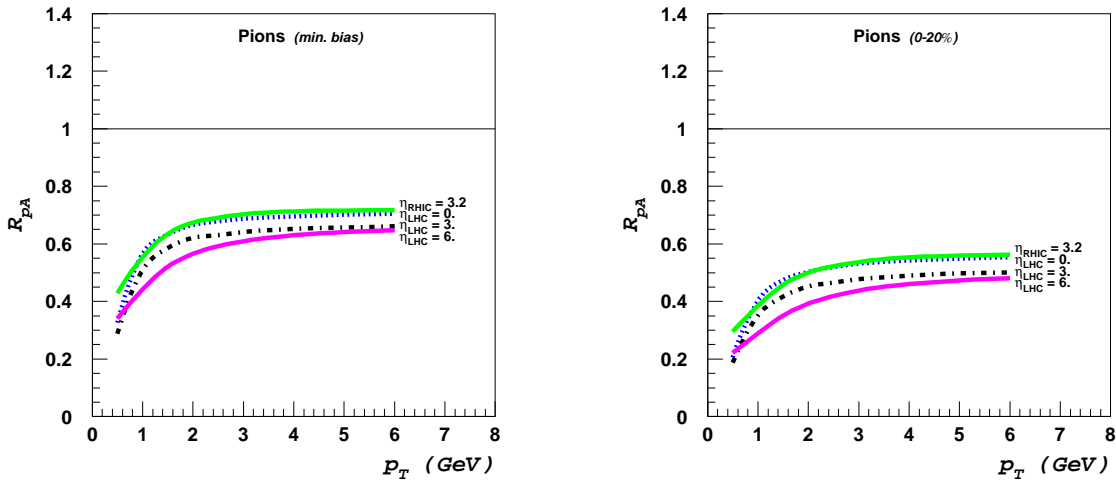


FIG. 1: Nuclear modification factor for pion production at RHIC and LHC.

out in [1], since rapidity  $\eta = 3.2$  at RHIC corresponds to almost the same  $x$  as  $\eta = 0$  at LHC, the nuclear modification factor is expected to be the same. This is seen in Fig. 1. We also observe that the difference between  $\eta = 0$  and  $\eta = 6$  is less than 20%. This teaches us that fast onset of the gluon saturation observed at RHIC is replaced by virtually  $y$ -independent behavior at LHC (from  $\eta = 0$  to  $\eta = 6$ ). Behavior of  $R_{pA}$  at rapidity  $\eta_{\text{LHC}} = -3.3$  and rapidity  $\eta_{\text{RHIC}} = 0$  is expected to be similar. The amount of suppression perceived in Fig. 1 depends mostly on centrality, i.e.  $N_{\text{coll}}$ :  $R_{pA} \simeq A^{-1/6} \simeq N_{\text{coll}}^{-1/2}$ . This yields  $R_{pA} = 0.60$  for minimal bias events and  $R_{pA} = 0.46$  for 0–20% centrality cut in agreement with Fig. 1.

To conclude this section we would like to emphasize that we explicitly neglect a possible effect of gluon saturation in a proton or deuteron. This approximation is *perhaps* sufficiently good for the nuclear modification factor which involves the ratio of the cross sections and is



intentionally constructed to look for nuclear effects.

#### IV. HEAVY QUARK PRODUCTION

Production of heavy quarks at small  $x$  is also affected by gluon saturation in a way similar to that of gluons [10]. The main difference, however, is that the effect of gluon saturation is postponed to higher energies/rapidities for heavier quarks as compared to lighter quarks and gluons. This is because the relevant  $x$  is proportional to  $m_\perp \sim (m^2 + k_\perp^2)^{1/2}$  and hence is higher for heavier quarks at the same values of  $\sqrt{s}, y, k$ .

In this section, we first of all review the theoretical approach to the  $q\bar{q}$  pair production at small  $x$ , see Ref. [7, 9]. Let us introduce the following notations:  $\underline{k}$  is the produced quark transverse momentum,  $\underline{q}$  is the gluon transverse momentum,  $\alpha = k_+/q_+$  is a fraction of the light-cone momentum of gluon carried by the produced quark;  $\underline{x}_1$  and  $\underline{y}_1$  are the transverse coordinates of the produced quark in the amplitude and in the complex conjugated amplitude respectively;  $\underline{x}_2$  and  $\underline{y}_2$  are the corresponding coordinates of the antiquark. Transverse coordinates of gluon in the amplitude  $\underline{u}$  and in the complex conjugated amplitude  $\underline{v}$  are given by  $\underline{u} \equiv \alpha \underline{x}_1 + (1 - \alpha) \underline{x}_2$  with  $u = |\underline{u}|$  and  $\underline{x}_{12} = \underline{x}_1 - \underline{x}_2$ , ( $x_{12} = |\underline{x}_{12}|$ ) and analogously for  $\underline{v}$ . With these notations the single inclusive quark production cross section is given by [7, 9]

$$\frac{d\sigma}{d^2k dy d^2b} = \frac{1}{2(2\pi)^4} \int d^2x_1 d^2x_2 d^2y_1 d^2y_2 \delta(\underline{u} - \underline{v} - \alpha(\underline{x}_{12} - \underline{y}_{12})) \int_0^1 d\alpha e^{-i\underline{k} \cdot (\underline{x}_1 - \underline{y}_1)} \quad (12)$$

$$\times \sum_{i,j=1}^3 \Phi_{ij}(\underline{x}_1, \underline{x}_2; \underline{y}_1, \underline{y}_2; \alpha) \Xi_{ij}(\underline{x}_1, \underline{x}_2; \underline{y}_1, \underline{y}_2; \alpha), \quad (13)$$

where the delta function comes about after integration over the antiquark transverse momentum which implies  $\underline{x}_2 = \underline{y}_2$ . The products of the light-cone “wave functions” are detailed as follows [9] see Fig. 2

$$\begin{aligned} \Phi_{11}(\underline{x}_1, \underline{x}_2; \underline{y}_1, \underline{y}_2; \alpha) &= 4C_F \left( \frac{\alpha_s}{\pi} \right)^2 \left\{ F_2(\underline{x}_1, \underline{x}_2; \alpha) F_2(\underline{y}_1, \underline{y}_2; \alpha) \frac{1}{x_{12} y_{12} u v} [(1 - 2\alpha)^2 \right. \\ &\quad \times (\underline{x}_{12} \cdot \underline{u}) (\underline{y}_{12} \cdot \underline{v}) + (\epsilon_{ij} u_i x_{12j}) (\epsilon_{kl} v_k y_{12l})] + F_1(\underline{x}_1, \underline{x}_2; \alpha) F_1(\underline{y}_1, \underline{y}_2; \alpha) m^2 \frac{\underline{u} \cdot \underline{v}}{u v} \\ &\quad \left. + 4\alpha^2 (1 - \alpha)^2 F_0(\underline{x}_1, \underline{x}_2; \alpha) F_0(\underline{y}_1, \underline{y}_2; \alpha) - 2\alpha (1 - \alpha) (1 - 2\alpha) \left[ \frac{\underline{x}_{12} \cdot \underline{u}}{x_{12} u} F_2(\underline{x}_1, \underline{x}_2; \alpha) \right. \right. \end{aligned}$$

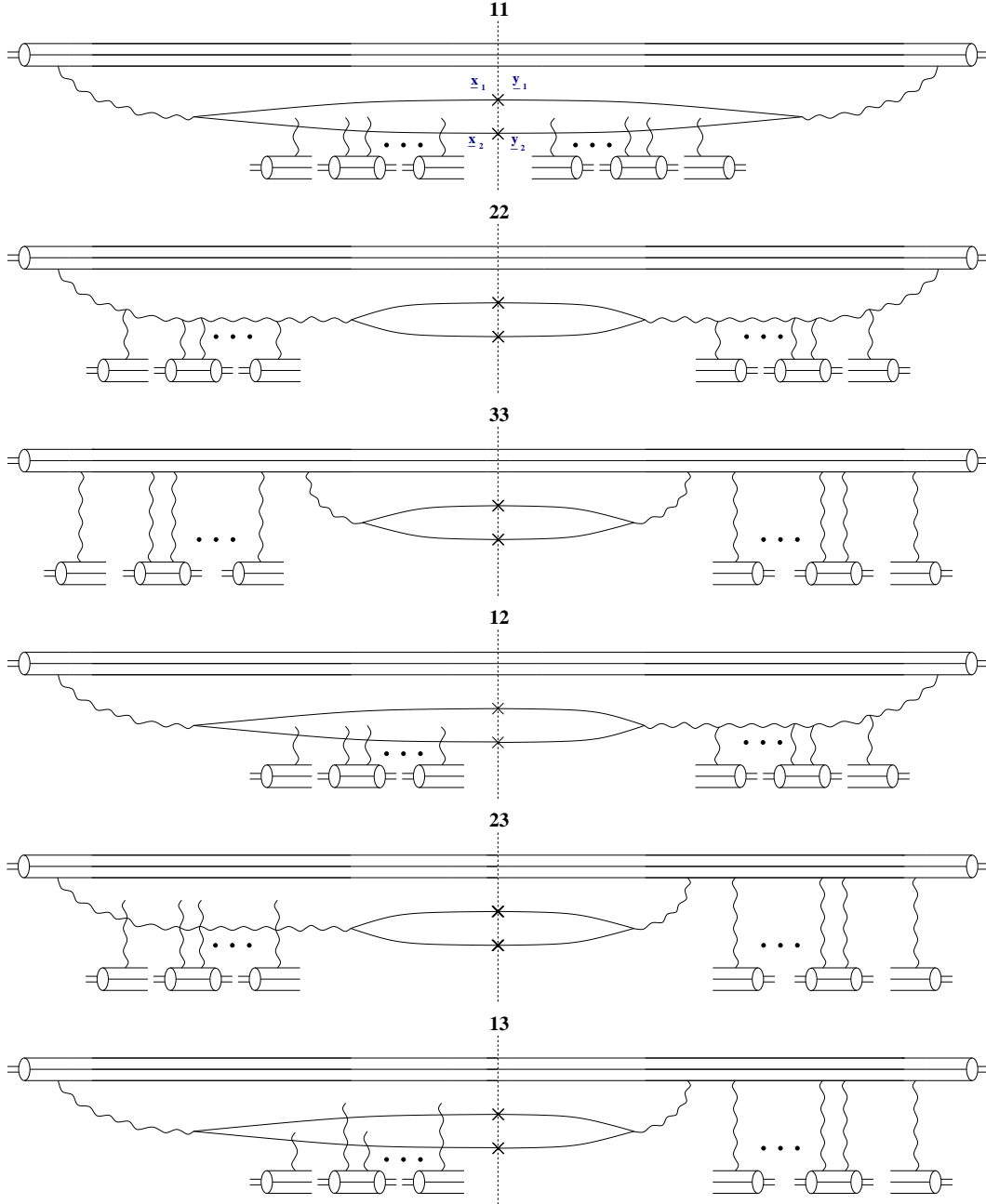


FIG. 2: The diagrams contributing to the quark–anti-quark pair production in the quasi-classical approximation. Disconnected  $t$ -channel gluon lines imply summation over all possible connections to the adjacent  $s$ -channel quark and gluon lines. Adopted from [9].

$$\times \left. F_0(\underline{y}_1, \underline{y}_2; \alpha) + \frac{\underline{y}_{12} \cdot \underline{v}}{y_{12} v} F_2(\underline{y}_1, \underline{y}_2; \alpha) F_0(\underline{x}_1, \underline{x}_2; \alpha) \right] \Bigg\}, \quad (14a)$$

$$\Phi_{22}(\underline{x}_1, \underline{x}_2; \underline{y}_1, \underline{y}_2; \alpha) = 4 C_F \left( \frac{\alpha_s}{\pi} \right)^2 m^2 \left\{ K_1(m x_{12}) K_1(m y_{12}) \frac{1}{x_{12} y_{12} u^2 v^2} [(1 - 2\alpha)^2 \right.$$

$$\times (\underline{x}_{12} \cdot \underline{u}) (\underline{y}_{12} \cdot \underline{v}) + (\epsilon_{ij} u_i x_{12j}) (\epsilon_{kl} v_k y_{12l})] + K_0(m x_{12}) K_0(m y_{12}) \frac{\underline{u} \cdot \underline{v}}{u^2 v^2} \Big\}, \quad (14b)$$

$$\begin{aligned} \Phi_{12}(\underline{x}_1, \underline{x}_2; \underline{y}_1, \underline{y}_2; \alpha) = & -4 C_F \left( \frac{\alpha_s}{\pi} \right)^2 m \left\{ F_2(\underline{x}_1, \underline{x}_2; \alpha) K_1(m y_{12}) \frac{1}{x_{12} y_{12} u v^2} [(1 - 2\alpha)^2 \right. \\ & \times (\underline{x}_{12} \cdot \underline{u}) (\underline{y}_{12} \cdot \underline{v}) + (\epsilon_{ij} u_i x_{12j}) (\epsilon_{kl} v_k y_{12l})] + m F_1(\underline{x}_1, \underline{x}_2; \alpha) K_0(m y_{12}) \frac{\underline{u} \cdot \underline{v}}{u v^2} \\ & \left. - 2\alpha(1 - \alpha)(1 - 2\alpha) \frac{y_{12} \cdot v}{y_{12} v^2} F_0(\underline{x}_1, \underline{x}_2; \alpha) K_1(m y_{12}) \right\}, \end{aligned} \quad (14c)$$

$$\begin{aligned} \Phi_{33}(\underline{x}_1, \underline{x}_2; \underline{y}_1, \underline{y}_2; \alpha) = & \Phi_{11}(\underline{x}_1, \underline{x}_2; \underline{y}_1, \underline{y}_2; \alpha) + \Phi_{22}(\underline{x}_1, \underline{x}_2; \underline{y}_1, \underline{y}_2; \alpha) + \Phi_{12}(\underline{x}_1, \underline{x}_2; \underline{y}_1, \underline{y}_2; \alpha) \\ & + \Phi_{21}(\underline{x}_1, \underline{x}_2; \underline{y}_1, \underline{y}_2; \alpha) \end{aligned} \quad (14d)$$

$$\Phi_{13}(\underline{x}_1, \underline{x}_2; \underline{y}_1, \underline{y}_2; \alpha) = -\Phi_{11}(\underline{x}_1, \underline{x}_2; \underline{y}_1, \underline{y}_2; \alpha) - \Phi_{12}(\underline{x}_1, \underline{x}_2; \underline{y}_1, \underline{y}_2; \alpha) \quad (14e)$$

$$\Phi_{23}(\underline{x}_1, \underline{x}_2; \underline{y}_1, \underline{y}_2; \alpha) = -\Phi_{21}(\underline{x}_1, \underline{x}_2; \underline{y}_1, \underline{y}_2; \alpha) - \Phi_{22}(\underline{x}_1, \underline{x}_2; \underline{y}_1, \underline{y}_2; \alpha) \quad (14f)$$

$$\Phi_{ij}(\underline{x}_1, \underline{x}_2; \underline{y}_1, \underline{y}_2; \alpha) = \Phi_{ji}^*(\underline{y}_1, \underline{y}_2; \underline{x}_1, \underline{x}_2; \alpha). \quad (14g)$$

The auxiliary functions  $F_1$ ,  $F_2$  and  $F_0$  are defined as

$$F_2(\underline{x}_1, \underline{x}_2; \alpha) = \int_0^\infty dq J_1(q u) K_1\left(x_{12} \sqrt{m^2 + q^2 \alpha(1 - \alpha)}\right) \sqrt{m^2 + q^2 \alpha(1 - \alpha)}, \quad (15a)$$

$$F_1(\underline{x}_1, \underline{x}_2; \alpha) = \int_0^\infty dq J_1(q u) K_0\left(x_{12} \sqrt{m^2 + q^2 \alpha(1 - \alpha)}\right), \quad (15b)$$

$$F_0(\underline{x}_1, \underline{x}_2; \alpha) = \int_0^\infty dq q J_0(q u) K_0\left(x_{12} \sqrt{m^2 + q^2 \alpha(1 - \alpha)}\right), \quad (15c)$$

where  $u = |\underline{u}|$ ,  $\underline{x}_{12} = \underline{x}_1 - \underline{x}_2$ ,  $x_{12} = |\underline{x}_{12}|$ , and  $\underline{q} = \underline{k}_1 + \underline{k}_2$ .

Using the definition of the *gluon* saturation scale  $Q_s$  (see (4))

$$Q_s^2 = 4\pi \alpha_s^2 \rho T(\underline{b}) \quad (16)$$

with  $\rho$  the nucleon number density in the nucleus and  $T(\underline{b})$  the nuclear profile function, we write (in the large  $N_c$  approximation)

$$\Xi_{11}(\underline{x}_1, \underline{x}_2; \underline{y}_1, \underline{y}_2; \alpha) = e^{-\frac{1}{8}(\underline{x}_1 - \underline{y}_1)^2 Q_s^2 \ln(1/|\underline{x}_1 - \underline{y}_1| \mu) - \frac{1}{8}(\underline{x}_2 - \underline{y}_2)^2 Q_s^2 \ln(1/|\underline{x}_2 - \underline{y}_2| \mu)}, \quad (17a)$$

$$\Xi_{22}(\underline{x}_1, \underline{x}_2; \underline{y}_1, \underline{y}_2; \alpha) = e^{-\frac{1}{4}(\underline{u} - \underline{v})^2 Q_s^2 \ln(1/|\underline{u} - \underline{v}| \mu)}, \quad (17b)$$

$$\Xi_{33}(\underline{x}_1, \underline{x}_2; \underline{y}_1, \underline{y}_2; \alpha) = 1, \quad (17c)$$

$$\Xi_{12}(\underline{x}_1, \underline{x}_2; \underline{y}_1, \underline{y}_2; \alpha) = e^{-\frac{1}{8}(\underline{x}_1 - \underline{v})^2 Q_s^2 \ln(1/|\underline{x}_1 - \underline{v}| \mu) - \frac{1}{8}(\underline{x}_2 - \underline{v})^2 Q_s^2 \ln(1/|\underline{x}_2 - \underline{v}| \mu)}, \quad (17d)$$

$$\Xi_{23}(\underline{x}_1, \underline{x}_2; \underline{y}_1, \underline{y}_2; \alpha) = e^{-\frac{1}{4}u^2 Q_s^2 \ln(1/u \mu)}, \quad (17e)$$

$$\Xi_{13}(\underline{x}_1, \underline{x}_2; \underline{y}_1, \underline{y}_2; \alpha) = e^{-\frac{1}{8}x_1^2 Q_s^2 \ln(1/x_1 \mu) - \frac{1}{8}x_2^2 Q_s^2 \ln(1/x_2 \mu)} \quad (17f)$$

All other  $\Xi_{ij}$ 's can be found from the components listed in (17) using

$$\Xi_{ij}(\underline{x}_1, \underline{x}_2; \underline{y}_1, \underline{y}_2; \alpha) = \Xi_{ji}(\underline{y}_1, \underline{y}_2; \underline{x}_1, \underline{x}_2; \alpha) \quad (18)$$

similar to (14g).

If the typical gluon momentum  $\underline{q}$  is much smaller than the produced quark mass, the above expressions can be significantly simplified. Indeed, since  $\alpha(1 - \alpha) \leq 1/4$  we get

$$\underline{q}^2 \alpha(1 - \alpha) \ll m^2, \quad (19)$$

and the auxiliary functions read

$$F_2(\underline{x}_1, \underline{x}_2; \alpha) = K_1(x_{12}m) m u^{-1}, \quad (20a)$$

$$F_1(\underline{x}_1, \underline{x}_2; \alpha) = K_0(x_{12}m) u^{-1}, \quad (20b)$$

$$F_0(\underline{x}_1, \underline{x}_2; \alpha) = 0. \quad (20c)$$

In this approximation the only non-vanishing products of “wave functions” are given by

$$\begin{aligned} \Phi_{11}(\underline{x}_1, \underline{x}_2; \underline{y}_1, \underline{y}_2; \alpha) &= \Phi_{22}(\underline{x}_1, \underline{x}_2; \underline{y}_1, \underline{y}_2; \alpha) = -\Phi_{12}(\underline{x}_1, \underline{x}_2; \underline{y}_1, \underline{y}_2; \alpha) \\ &= 4 C_F \left( \frac{\alpha_s}{\pi} \right)^2 m^2 \left\{ K_1(x_{12}m) K_1(y_{12}m) \frac{1}{x_{12} y_{12} u^2 v^2} [(1 - 2\alpha)^2 (\underline{x}_{12} \cdot \underline{u}) (\underline{y}_{12} \cdot \underline{v}) \right. \\ &\quad \left. + (\epsilon_{ij} u_i x_{12j}) (\epsilon_{kl} v_k y_{12l})] + K_0(x_{12}m) K_0(y_{12}m) \frac{\underline{u} \cdot \underline{v}}{u^2 v^2} \right\}. \end{aligned}$$

Averaging over all directions of gluon emission from the valence quark using  $\langle \epsilon_{ij} u_i x_{12j} \epsilon_{kl} v_k y_{12l} \rangle = (1/2) u v \underline{x}_{12} \cdot \underline{y}_{12}$  we arrive at the well-known result [7, 37]

$$\begin{aligned} \Phi_{11}(\underline{x}_1, \underline{x}_2; \underline{y}_1, \underline{y}_2; \alpha) &= \\ 4 C_F \left( \frac{\alpha_s}{\pi} \right)^2 \frac{m^2}{uv} &\left\{ \frac{\underline{x}_{12} \cdot \underline{y}_{12}}{x_{12} y_{12}} [(1 - \alpha^2) + \alpha^2] K_1(x_{12}m) K_1(y_{12}m) + K_0(x_{12}m) K_0(y_{12}m) \right\} \quad (21) \end{aligned}$$

Approximation (19), employed to derive (21), amounts to the  $k_T$ -factorization of the gluon distribution of proton (see (24)). It breaks down if  $x$  is so small that proton can no longer be regarded as a dilute object: it acquires a hard scale  $\underline{q}^2 \sim Q_{sp}^2$  which increases with energy. However, in such a case even a more general formulas (14), (17) which go beyond the  $k_T$ -factorization of proton by do not include the high density effects, become invalid too

as they neglect the *nonlinear* gluon evolution in proton. Discussion of the gluon saturation in proton is beyond the scope of this paper.

It is convenient to express the scattering amplitude in terms of the vectors  $\underline{x}_{12}$ ,  $\underline{y}_{12}$ ,  $\underline{u}$  and  $\underline{v}$  and integrate in (13) over these variables. We have

$$\underline{x}_1 - \underline{y}_1 = \underline{x}_{12} - \underline{y}_{12}, \quad (22a)$$

$$\underline{x}_1 - \underline{y}_1 = 0, \quad (22b)$$

$$\underline{x}_1 - \underline{v} = \underline{x}_{12} - \alpha \underline{y}_{12}, \quad (22c)$$

$$\underline{x}_2 - \underline{v} = -\alpha \underline{y}_{12}, \quad (22d)$$

$$\underline{y}_1 - \underline{u} = -\alpha \underline{x}_{12} + \underline{y}_{12}, \quad (22e)$$

$$\underline{y}_2 - \underline{u} = -\alpha \underline{x}_{12}. \quad (22f)$$

Thus, sum over all rescattering factors including the signs of  $\Phi_{ij}$ 's is given by (we omit logarithms  $\ln(1/x\mu)$  for brevity)

$$\begin{aligned} \Xi(\underline{x}_{12}, \underline{y}_{12}; \alpha) = & e^{-\frac{1}{4}(\underline{x}_{12}-\underline{y}_{12})^2(Q_s^2/2)} + e^{-\frac{1}{2}\alpha^2(\underline{x}_{12}-\underline{y}_{12})^2(Q_s^2/2)} \\ & - e^{-\frac{1}{4}(\underline{x}_{12}-\alpha\underline{y}_{12})^2(Q_s^2/2)} e^{-\frac{1}{4}\alpha^2 y^2(Q_s^2/2)} - e^{-\frac{1}{4}(\alpha\underline{x}_{12}-\underline{y}_{12})^2(Q_s^2/2)} e^{-\frac{1}{4}\alpha^2 x_{12}^2(Q_s^2/2)}. \end{aligned} \quad (23)$$

Using the delta function in (13) to integrate over  $\underline{u}$  and integrating over  $\underline{v}$  in the leading logarithmic approximation we derive the final result

$$\begin{aligned} \frac{d\sigma}{d^2k dy d^2b} = & \frac{C_F \alpha_s^2 m^2}{4\pi^5} \int_0^1 d\alpha \int d^2x_{12} d^2y_{12} e^{-ik \cdot (\underline{x}_{12} - \underline{y}_{12})} \ln(1/\mu |\underline{x}_{12} - \underline{y}_{12}|) \\ & \times \left\{ \frac{\underline{x}_{12} \cdot \underline{y}_{12}}{x_{12} y_{12}} [(1 - \alpha^2) + \alpha^2] K_1(x_{12}m) K_1(y_{12}m) + K_0(x_{12}m) K_0(y_{12}m) \right\} \Xi(\underline{x}_{12}, \underline{y}_{12}; \alpha) \end{aligned} \quad (24)$$

Before we turn to the numerical results, we would like to remark on the large  $N_c$  corrections to (24). In our approach these corrections arise in the scattering amplitudes (17). Exact expressions for those amplitudes in the quasi-classical approximation were derived in [7, 8]. Numerical uncertainty due to omission of the  $\sim 1/N_c^2$  terms is expected to be of the order of 10%. A detailed analysis performed in [39] suggests that these corrections are even smaller. Another source of the large  $N_c$  corrections is the high energy gluon evolution beyond the mean-field approximation. These corrections were studied in detail in [40] and are shown to be at the 1% level. Alluding to the poorly known NLO corrections as well as the accuracy of the present experimental data, we think that neglecting the large  $N_c$  corrections is justified.

Numerical calculations are performed along the same steps as in Sec. III: we replace the dipole scattering amplitude by the model (1) and (5) and take into account the kinematic factors (10), (11). In (2) and (3) we substitute  $\underline{r}^2 \approx 1/(4m_\perp^2)$ . To obtain spectra of D and B mesons we convolute the cross section (24) with the Peterson fragmentation function  $D(z)$

$$\frac{d\sigma_{\text{hadron}}}{d^2p dy} = \int_{z_{\min}}^1 \int d^2k \frac{d\sigma(\underline{k}, y)}{d^2k dy} \delta(\underline{p} - z\underline{k}) D(z) = \int_{z_{\min}}^1 \frac{dz}{z^2} \frac{d\sigma(\underline{p}/z, y)}{d^2k dy} D(z), \quad (25)$$

where  $\underline{p}$  is the hadron's transverse momentum and

$$D(z) \propto \frac{1}{z} \left( 1 - \frac{1}{z} - \frac{\epsilon}{1-z} \right)^{-2}, \quad (26)$$

with  $\epsilon = 0.043$  for c-quark and  $\epsilon = 0.006$  for b-quark. The results of the calculations are displayed in Fig. 3 and Fig. 4.

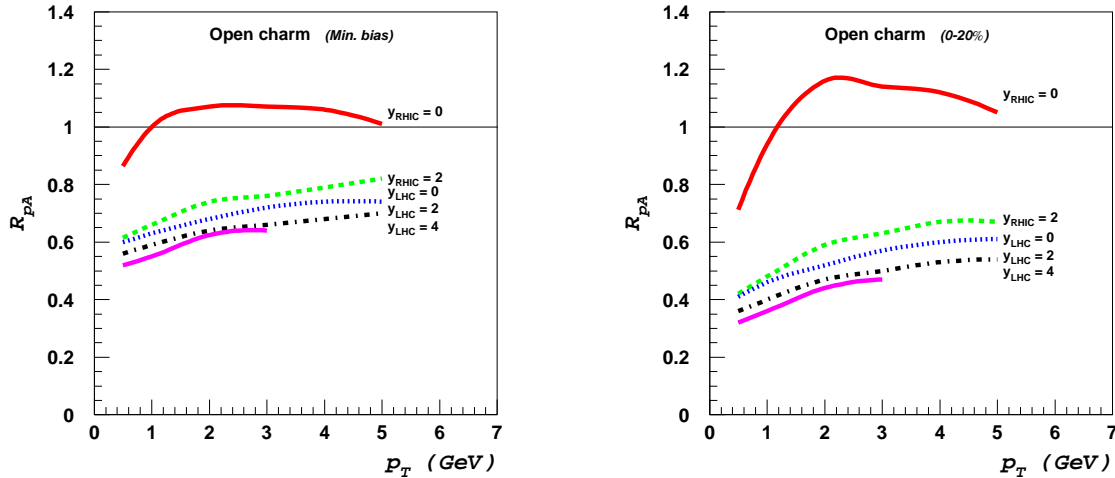


FIG. 3: Nuclear modification factor for open charm production at RHIC and LHC.

Note, that the calculations of [7, 9] are applicable only if the coherence length of the typical  $q\bar{q}$  pair is much larger than the size interaction of the interaction region which, in the nucleus rest frame, is about the size of the nucleus for the most central events. It is easy to verify (see e. g. [38]) that this condition holds for all lines in Fig. 3 and Fig. 4 apart from the B-meson production at  $y = 0$  at RHIC. In the former case, the hadron nuclear attenuation effect can be responsible for suppression (not show here) of B production.

In this paper we used a model for  $N(\underline{r}, y)$  which was tested in the RHIC kinematics. Extension to LHC involves an assumption that this model correctly captures the energy

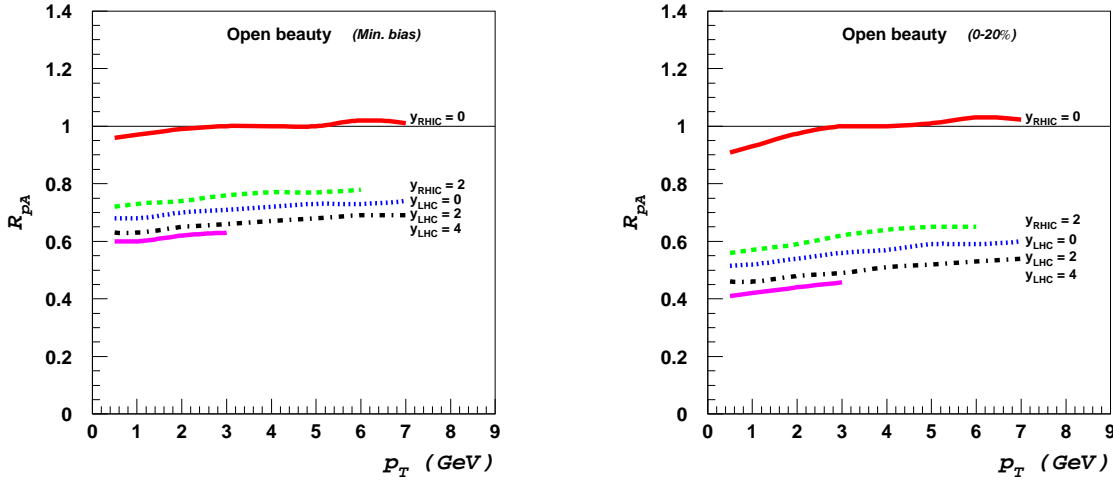


FIG. 4: Nuclear modification factor for open beauty production at RHIC and LHC.

dependence of the inclusive cross sections. It is of great interest to look in yet another kinematic region of low  $x$  and high  $m_{\perp}$ . The very first paper on geometric scaling [41] hints that the geometric scaling holds in quite a wide kinematic region of  $x < 0.01$  and  $Q^2 \leq 450 \text{ GeV}^2$  (and all small  $x$  data available). It was explained in [42] that this property stems directly from solution to BK equation. The geometric scaling is predicted to hold up to momenta  $Q_{\text{geom}} \sim Q_s^2/\Lambda$  [43]. Although there is a fair amount of ambiguity in the value of  $\Lambda$  we can use the model of Sec. II to estimate that at LHC at  $y = 0$   $Q_{\text{geom}} = 2.7 \text{ GeV}$ , and at  $y = 2$   $Q_{\text{geom}} = 5.0 \text{ GeV}$ . On the other hand, the original observation of [41] as well as the parameterization of [1] seems to imply that the geometric scaling may still be a reasonable approximation at even higher momenta, provided that  $x$  is small enough. In order to illustrate dependence of  $R_{pA}$  on  $m_{\perp}$  we calculated it for four quark flavors shown in Fig. 5. At masses of the order of  $m_t$  and at rapidity  $y = 2$  at LHC we have  $x \simeq 0.01$ , so that  $x$  is indeed small enough for the dipole model to be applicable. The amount of suppression of  $R_{pA}$  depends on the anomalous dimension  $\gamma$  as, roughly  $R_{pA} \sim 1/N_{\text{coll}}^{1-\gamma}$ , where  $\gamma$  is a function of  $y$  and  $m_{\perp}$ , see (2) and (3). Therefore, by measuring the  $m_{\perp}$  and  $y$  dependence of  $R_{pA}$  one can determine the anomalous dimension  $\gamma$  and map the transition region between the geometric scaling and the collinear factorization. The failure of the geometric scaling will manifest itself by rapid approach of the  $R_{pA}$  to unity (modulo DGLAP corrections) at

transverse mass  $m_{\perp} \gtrsim Q_{\text{geom}}$ .

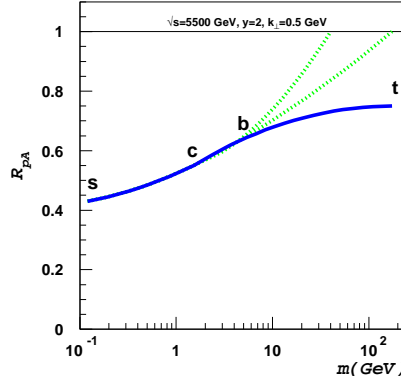


FIG. 5: Dependence of the nuclear modification factor on quark mass. Solid line is  $R_{pA}$  for quarks (no fragmentation). Geometric scaling is expected to break down at  $m_{\perp} \sim Q_{\text{geom}}$  and therefore  $R_{pA}$  is anticipated to deviate from the solid line towards unity. Dotted lines illustrates a possible behavior of  $R_{pA}$ .

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